International Journal of Engineering, Science and Mathematics
Vol. 9 Issue 6, June 2020,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

ON FIXED POINT RESULTS IN COMPLETE G-METRIC SPACES

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ABSTRACT

In this paper,we study some existence and uniqueness fixed point theorems in complete G-metric space using a new generalized weakly contractive condition which improve and generalize some metric fixed piont results in the literature.

*Keywords**:** G-metric space ; Fixed point; Lower semi continuity; Generalized weakly contraction mapping .*

1. INTRODUCTION AND PRELIMINARIES

In 2006, Mustafa and Sims [1] introduced a notion of generalized metric space called G-metric space . Afterwards, several authors studied many fixed point results for self-mappings in G-metric spaces under certain contractive conditions [2,3].

We now present some preliminaries and definitions which are used in the sequel.

Definition 1.1. **[1]** Let X be a non empty set and let $G: X \times X \times X \to [0, \infty)$ be a function satisfying the following properties:

$G(x, y, z) = 0$ if $x = y = z$.

$G(x, x, y) > 0$ for all $x, y \in X$, with $x \neq y$.

$G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$, with $y \neq z$.

$G(x, y, z) = G(x, z, y) = G(y, z, x) = \ldots$, (symmetry in all three variables)

$G(x, y, z) \leq G(x, a, a) + G(a, y, z)$, for all $x, y, z, a \in X$ (rectangular inequality)

Then the function G is called a G-metric on X and the pair (X, G) is called a G-metric space.

Definition 1.2. **[2]** Let (X, G) be a G-metric space and let $\{x_n\}$ be a sequence of points in X. We say that $\{x_n\}$ is G-convergent to x if $\lim_{n,m\to\infty} G(x, x_n, x_m) = 0$. That is,for any $\varepsilon > 0$, there exist n∈N such that $G(x, x_n, x_m) < \varepsilon$ for all $n, m \geq$ n. We call x as the limit of the sequence $\{x_n\}$ and we write $x_n \to x$ as $n \to \infty$ or $\lim_{n\to\infty} x_n = x$.

Definition 1.3.**[1]** Let (X, G) be a G-metric space. A sequence $\{x_n\}$ is called G-Cauchy sequence if given $\varepsilon > 0$, there is N∈ n such that $G(x_n, x_m, x_l) < \varepsilon$ for all $n, m, l \geq$N, that is, $G(x_n, x_m, x_l) \to 0$ as $n, m, l \to \infty$.

Definition1.4.**[1]** A G-metric space (X, G) is said to be G-complete if every G-cauchy sequence in (X, G) is G-convergent in (X, G).

Proposition 1.5. **[1]** Let (X, G) be a G-metric space. Then for any $x, y, z, a \in X$, the following holds.

If $G(x, y, z) = 0$ then $x = y = z$.

Definition 1.6.**[4]** Let X be a metric space. A function $f: X \to [0, \infty)$ is called lower semi-continuous if $x \in X$ and $\{x_n\}$ in X with $\lim_{n\to\infty} x_n = x$, then $f(x) \leq \liminf_{n\to\infty} f(x_n)$.

Definition 1.7 **.[4]** Let (X, d) be a metric space and let $\varphi: X \to [0, \infty)$ be a lower semi-continuous function. A self map $T: X \to X$ is called a a generalized weakly contractive mapping if $\psi\big(d(Tx, Ty)\big) + \varphi(Tx) + \varphi(Ty) \leq \psi\big(M(x, y)\big) - \varphi(L(x, y))$ for all x,y $\in X$, where $\psi : [0, \infty) \to [0, \infty)$ is continuous with $\psi(t) = 0 \; iff \; t = 0$, $\varphi : [0, \infty) \to [0, \infty)$ is lower semi-continuous with $\varphi(t) = 0 \; iff \; t = 0$ and $M(x, y) = \max\{d(x, y) + \varphi(x) + \varphi(y), d(x, Tx) + \varphi(x) + \varphi(Tx), d(y, Ty) + \varphi(y) + \varphi(Ty), \frac{1}{2}(d(x, Ty) + \varphi(x) + \varphi(Ty) + d(y, Tx) + \varphi(y) + \varphi(Tx))\}$ and $L(x, y) = \max\{d(x, y) + \varphi(x) + \varphi(y), d(y, Ty) + \varphi(y) + \varphi(Ty)\}$.

In 2018, Seonghoon Cho proved the following theorem .

Theorem 1.10. **[4]** Let (X , d) be a complete metric space . If T is a generalized weakly contractive mapping , then there exists a unique z in X such that z = Tz and $\varphi(z) = 0$.

We now prove a theorem analogous to the above theorem.

2. MAIN RESULTS

Theorem 2.1. Let (X,G) be a complete G-metric space and T be a self map of X satisfying

$\mu\big(G(Tx,Ty,Tz)+\varphi(Tx)+\varphi(Ty)+\varphi(Tz)\big)\le\mu\big(L(x,y,z)\big)-\varphi\big(M(x,y,z)\big)$ for all x,y,z in X (1) where $L(x,y,z)=\max\{G(x,y,z)+\varphi(x)+\varphi(y)+\varphi(z), G(x,Tx,Tx)+\varphi(x)+2\varphi(Tx), G(y,Ty,Ty)+\varphi(y)+2\varphi(Ty), G(z,Tz,Tz)+\varphi(z)+2\varphi(Tz), \frac{1}{3}\{G(x,Ty,Ty)+\varphi(x)+2\varphi(Ty)+G(y,Tz,Tz)+\varphi(y)+2\varphi(Tz)+G(z,Tx,Tx)+\varphi(z)+2\varphi(Tx)\}$, and $M(x,y,z)=\max\{G(x,y,z)+\varphi(x)+\varphi(y)+\varphi(z), G(y,Ty,Ty)+\varphi(y)+2\varphi(Ty), G(z,Tz,Tz)+\varphi(z)+2\varphi(Tz)\}$ and $\varphi:[0,\infty)\to[0,\infty)$ is a lower semi-continuous function with $\varphi(t)=0\ iff\ t=0$ and $\mu:[0,\infty)\to[0,\infty)$ is continuous such that $\mu(t)=0$ iff $t=0$. Then there exists a unique p in X such that p = Tp and $\varphi(p)=0$.

Proof : Let $x_0\in X$ be arbitrary . Define a sequence $\{x_n\}$ in X such that $x_n=Tx_{n-1}$ for all $n\ge1$. If $x_n=x_{n-1}$ for some n, then T has a fixed point. Assume that $x_n\ne x_{n-1}$ for all $n\ge1$. Now, for all $n\ge1$ from (1) we have

$\mu\big(G(Tx_n,Tx_{n+1},Tx_{n+1})+\varphi(Tx_n)+2\varphi(Tx_{n+1})\big)\le\mu\big(L(x_n,x_{n+1},x_{n+1})\big)-\varphi\big(M(x_n,x_{n+1},x_{n+1})\big)$. (2) where, $L(x_n,x_{n+1},x_{n+1})=\max\Big\{G(x_n,x_{n+1},x_{n+1})+\varphi(x_n)+2\varphi(x_{n+1}), G(x_n,Tx_n,Tx_n)+\varphi(x_n)+2\varphi(Tx_n), G(x_{n+1},Tx_{n+1},Tx_{n+1})+\varphi(x_{n+1})+2\varphi(Tx_{n+1}),$

$G(x_{n+1},Tx_{n+1},Tx_{n+1})+\varphi(x_{n+1})+2\varphi(Tx_{n+1}), \frac{1}{3}\big(G(x_n,Tx_{n+1},Tx_{n+1})+\varphi(x_n)+2\varphi(Tx_{n+1})+G(x_{n+1},Tx_{n+1},Tx_{n+1})+\varphi(x_{n+1})+2\varphi(Tx_{n+1})+G(x_{n+1},Tx_n,Tx_n)+\varphi(x_{n+1})+2\varphi(Tx_n)\big)\Big\}$

But $\frac{1}{3}\big(G(x_n,Tx_{n+1},Tx_{n+1})+\varphi(x_n)+2\varphi(Tx_{n+1})+G(x_{n+1},Tx_{n+1},Tx_{n+1})+\varphi(x_{n+1})+2\varphi(Tx_{n+1})+G(x_{n+1},Tx_n,Tx_n)+\varphi(x_{n+1})+2\varphi(Tx_n)\big)=$
$\frac{1}{3}\big(G(x_n,x_{n+2},x_{n+2})+\varphi(x_n)+2\varphi(x_{n+2})+G(x_{n+1},x_{n+2},x_{n+2})+\varphi(x_{n+1})+2\varphi(x_{n+2})+G(x_{n+1},x_{n+1},x_{n+1})+\varphi(x_{n+1})+2\varphi(x_{n+1})\big)\le\frac{1}{3}\big(G(x_n,x_{n+1},x_{n+1})+G(x_{n+1},x_{n+2},x_{n+2})+\varphi(x_n)+2\varphi(x_{n+2})+G(x_{n+1},x_{n+2},x_{n+2})+\varphi(x_{n+1})+2\varphi(x_{n+2})+G(x_{n+1},x_{n+1},x_{n+1})+\varphi(x_{n+1})+2\varphi(x_{n+1})\big)$

$\leq$ max{ $G(x_n, x_{n+1}, x_{n+1}) + \varphi(x_n) + 2\varphi(x_{n+1}), G(x_{n+1}, x_{n+2}, x_{n+2}) + \varphi(x_{n+1}) + 2\varphi(x_{n+2})\}$ Hence $L(x_n, x_{n+1}, x_{n+1}) = \max\{G(x_n, x_{n+1}, x_{n+1}) + \varphi(x_n) + 2\varphi(x_{n+1}),$ $G(x_{n+1}, x_{n+2}, x_{n+2}) + \varphi(x_{n+1}) + 2\varphi(x_{n+2})\}$. Now

$M(x_n, x_{n+1}, x_{n+1}) = \max\{G(x_n, x_{n+1}, x_{n+1}) + \varphi(x_n) + 2\varphi(x_{n+1}),$ $G(x_{n+1}, x_{n+2}, x_{n+2}) + \varphi(x_{n+1}) + 2\varphi(x_{n+2}), G(x_{n+1}, x_{n+2}, x_{n+2}) + \varphi(x_{n+1}) + 2\varphi(x_{n+2})\}$

Suppose $G(x_n, x_{n+1}, x_{n+1}) + \varphi(x_n) + 2\varphi(x_{n+1}) < G(x_{n+1}, x_{n+2}, x_{n+2}) + \varphi(x_{n+1}) + 2\varphi(x_{n+2})$ for some integer n. Then from (1), we have $\mu\big(G(x_{n+1}, x_{n+2}, x_{n+2}) + \varphi(x_{n+1}) + 2\varphi(x_{n+2})\big) \leq \mu\big(G(x_{n+1}, x_{n+2}, x_{n+2}) + \varphi(x_{n+1}) + 2\varphi(x_{n+2})\big) - \varphi\big(G(x_{n+1}, x_{n+2}, x_{n+2}) + \varphi(x_{n+1}) + 2\varphi(x_{n+2})\big)$ which implies that $\varphi\big(G(x_{n+1}, x_{n+2}, x_{n+2}) + \varphi(x_{n+1}) + 2\varphi(x_{n+2})\big) = 0$ and hence $G(x_{n+1}, x_{n+2}, x_{n+2}) + \varphi(x_{n+1}) + 2\varphi(x_{n+2}) = 0$. So $\varphi(x_{n+1}) = \varphi(x_{n+2}) = 0$ and $x_{n+1} = x_{n+2}$, a contradiction. Thus $G(x_{n+1}, x_{n+2}, x_{n+2}) + \varphi(x_{n+1}) + 2\varphi(x_{n+2}) \leq G(x_n, x_{n+1}, x_{n+1}) + \varphi(x_n) + 2\varphi(x_{n+1})$ for all n , which shows that the seuqence $\{G(x_{n+1}, x_{n+2}, x_{n+2}) + \varphi(x_{n+1}) + 2\varphi(x_{n+2})\}$ is monotonic decreasing and hence converges to some real number $r \geq 0$. i.e., $\lim_{n\to\infty} G(x_{n+1}, x_{n+2}, x_{n+2}) + \varphi(x_{n+1}) + 2\varphi(x_{n+2}) = r$

(3)

So we have $L(x_n, x_{n+1}, x_{n+1}) = G(x_n, x_{n+1}, x_{n+1}) + \varphi(x_n) + 2\varphi(x_{n+1})$ and $M(x_n, x_{n+1}, x_{n+1}) = G(x_n, x_{n+1}, x_{n+1}) + \varphi(x_n) + 2\varphi(x_{n+1})$ for all n.

Suppose $r > 0$. Now from (1) we get that

$\mu\big(G(x_{n+1}, x_{n+2}, x_{n+2}) + \varphi(x_{n+1}) + 2\varphi(x_{n+2})\big) \leq \mu\big(G(x_n, x_{n+1}, x_{n+1}) + \varphi(x_n) + 2\varphi(x_{n+1})\big) - \varphi\big(G(x_n, x_{n+1}, x_{n+1}) + \varphi(x_n) + 2\varphi(x_{n+1})\big)$.

(1.1) Letting $n\to\infty$ and in view of the properties of μ, φ , we get $\mu(r) \leq \mu(r) - \varphi(r)$ implies $\varphi(r) \leq 0$, a contradiction. Hence r = 0. That is, $\lim_{n\to\infty} G(x_{n+1}, x_{n+2}, x_{n+2}) + \varphi(x_{n+1}) + 2\varphi(x_{n+2}) = 0$. So that $\lim_{n\to\infty} G(x_{n+1}, x_{n+2}, x_{n+2}) = 0$ and $\lim_{n\to\infty} \varphi(x_{n+1}) = 0$. (4)

(1.2) We now claim that the sequence $\{x_n\}$ is Cauchy. On the contrary , suppose that $\{x_n\}$ is not Cauchy. Then there exists an $\varepsilon > 0$ and two sequences $\{m(k)\}$ and $\{n(k)\}$ with n(k) > m(k) > k for all positive integers k such that $G\big(x_{m(k)}, x_{n(k)}, x_{n(k)}\big) \geq \varepsilon$ and $G\big(x_{m(k)}, x_{n(k)-1}, x_{n(k)-1}\big) < \varepsilon$.

On using rectangular inequality, (4) , (5) and letting $k\to\infty$,we obtain

$\varepsilon \le G(x_{m(k)}, x_{n(k)}, x_{n(k)}) \le G(x_{m(k)}, x_{n(k)-1}, x_{n(k)-1}) + G(x_{n(k)-1}, x_{n(k)}, x_{n(k)})$

So $\varepsilon \le G(x_{m(k)}, x_{n(k)}, x_{n(k)}) < \varepsilon + G(x_{m(k)-1}, x_{m(k)}, x_{n(k)})$.

$\varepsilon \le \lim_{k\to\infty} G(x_{m(k)}, x_{n(k)}, x_{n(k)}) < \varepsilon + 0$ and therefore $\lim_{k\to\infty} G(x_{m(k)}, x_{n(k)}, x_{n(k)}) = \varepsilon$.

(6)

Again by rectangular inequality, we have $G(x_{m(k)}, x_{n(k)}, x_{n(k)})$

$\le \{G(x_{m(k)}, x_{m(k)+1}, x_{m(k)+1})$

$+G(x_{m(k)+1}, x_{n(k)+1}, x_{n(k)+1}) +$

$G(x_{n(k)+1}, x_{n(k)}, x_{n(k)})\}$

and $G(x_{m(k)+1}, x_{n(k)+1}, x_{n(k)+1}) \le G(x_{m(k)+1}, x_{m(k)}, x_{m(k)})$

$+G(x_{m(k)}, x_{n(k)}, x_{n(k)}) + G(x_{n(k)}, x_{n(k)+1}, x_{n(k)+1})$

Letting k→∞, we get from (4) and (6) that $\lim_{k\to\infty} G(x_{m(k)+1}, x_{n(k)+1}, x_{n(k)+1}) = \varepsilon$.

(7)

Also $G(x_{m(k)}, x_{n(k)}, x_{n(k)}) \le G(x_{m(k)}, x_{n(k)+1}, x_{n(k)+1}) + G(x_{n(k)+1}, x_{n(k)}, x_{n(k)})$,

$G(x_{m(k)}, x_{n(k)+1}, x_{n(k)+1}) \le G(x_{m(k)}, x_{n(k)}, x_{n(k)}) + G(x_{n(k)}, x_{n(k)+1}, x_{n(k)+1})$.

Letting k→∞ in the above inequalities, we get $\lim_{k\to\infty} G(x_{m(k)}, x_{n(k)+1}, x_{n(k)+1}) = \varepsilon$.

(8)

Similarly, it can be shown that $\lim_{k\to\infty} G(x_{n(k)}, x_{m(k)+1}, x_{m(k)+1}) = \varepsilon$.

(9)

Now from (1) with $x = x_{m(k)},\ y = x_{n(k)},\ z = x_{n(k)}$, we have

$\mu\left(G(Tx_{m(k)}, Tx_{n(k)}, Tx_{n(k)})\right) + \varphi(Tx_{m(k)}) + 2\,\varphi(Tx_{n(k)}) \le$

$\mu(L(x_{m(k)}, x_{n(k)}, x_{n(k)})) - \varphi\left(M(x_{m(k)}, x_{n(k)}, x_{n(k)})\right)$

(10)

where

$L(x_{m(k)}, x_{n(k)}, x_{n(k)}) =$

$\max\Big\{G(x_{m(k)}, x_{n(k)}, x_{n(k)}) + \varphi(x_{m(k)}) + 2\varphi(x_{n(k)}), G(x_{m(k)}, Tx_{m(k)}, Tx_{m(k)}) +$

$\varphi(x_{m(k)}) + 2\varphi(Tx_{m(k)}), G(x_{n(k)}, Tx_{n(k)}, Tx_{n(k)}) + \varphi(x_{m(k)}) + 2\varphi(Tx_{n(k)}),$,

$\frac{1}{3}\Big(G(x_{m(k)}, Tx_{n(k)}, Tx_{n(k)}) + \varphi(x_{m(k)}) + 2\varphi(Tx_{n(k)}) + G(x_{n(k)}, Tx_{n(k)}, Tx_{n(k)}) +$

$\varphi(x_{n(k)}) + 2\varphi(Tx_{n(k)}) + G(x_{n(k)}, Tx_{m(k)}, Tx_{m(k)}) + \varphi(x_{n(k)}) + 2\varphi(Tx_{m(k)})\Big)\Big\} =$

$\max\Big\{G\big(x_{m(k)},x_{n(k)},x_{n(k)}\big)+\varphi\big(x_{m(k)}\big)+2\varphi\big(x_{n(k)}\big),G\big(x_{m(k)},x_{m(k)+1},x_{m(k)+1}\big)+$
$\varphi\big(x_{m(k)}\big)+2\varphi\big(x_{m(k)+1}\big),G\big(x_{n(k)},x_{n(k)+1},x_{n(k)+1}\big)+\varphi\big(x_{n(k)}\big)+2\varphi\big(x_{n(k)+1}\big),\ ,$
$\frac{1}{3}\Big(G\big(x_{m(k)},x_{n(k)+1},x_{n(k)+1}\big)+\varphi\big(x_{m(k)}\big)+2\varphi\big(x_{n(k)+1}\big)+G\big(x_{n(k)},x_{n(k)+1},x_{n(k)+1}\big)+$
$\varphi\big(x_{n(k)}\big)+2\varphi\big(x_{n(k)+1}\big)+G\big(x_{n(k)},x_{m(k)+1},x_{m(k)+1}\big)+\varphi\big(x_{n(k)}\big)+2\varphi\big(x_{m(k)+1}\big)\Big)\Big\}$

Letting k→∞, and using (4) to (9), we get $\lim_{k\to\infty} L\big(x_{m(k)},x_{n(k)},x_{n(k)}\big)=\varepsilon$.

Now,

$M\big(x_{m(k)},x_{n(k)},x_{n(k)}\big)=$
$\max\{G\big(x_{m(k)},x_{n(k)},x_{n(k)}\big)+\varphi\big(x_{m(k)}\big)+2\varphi\big(x_{n(k)}\big),G\big(x_{n(k)},Tx_{n(k)},Tx_{n(k)}\big)+$
$\varphi\big(x_{m(k)}\big)+2\varphi\big(Tx_{n(k)}\big),G\big(x_{n(k)},Tx_{n(k)},Tx_{n(k)}\big)+\varphi\big(x_{m(k)}\big)+2\varphi\big(Tx_{n(k)}\big)\}$

$$=\max\{G\big(x_{m(k)},x_{n(k)},x_{n(k)}\big)+\varphi\big(x_{m(k)}\big)+2\varphi\big(x_{n(k)}\big),G\big(x_{n(k)},x_{n(k)+1},x_{n(k)+1}\big)+\varphi\big(x_{n(k)}\big)+2\varphi\big(x_{n(k)+1}\big),G\big(x_{n(k)},x_{n(k)+1},x_{n(k)+1}\big)+\varphi\big(x_{n(k)}\big)+2\varphi\big(x_{n(k)+1}\big)\}$$

Letting k→∞, and using (4) to (9), we get $\lim_{k\to\infty} M\big(x_{m(k)},x_{n(k)},x_{n(k)}\big)=\varepsilon$.

Hence from (10) as k→∞ , we get $\mu(\varepsilon)\le\mu(\varepsilon)-\varphi(\varepsilon)$ implies $\varphi(\varepsilon)\le 0$, a contraction.(in view of the property of φ).Thus $\{x_n\}$ is a Cauchy sequence in X. Since X is a complete G-metric space, we can find a p∈ X such that $\lim_{n\to\infty} x_n$ = p. In view of the lower semi-continuity of φ , we have $\varphi(p)\le\liminf_{n\to\infty}\varphi(x_n)\le\lim_{n\to\infty}\varphi(x_n)=0$. That is, $\varphi(p)=0$.

We now prove that p is a fixed point of T.

On using (1) with x = x_n, y = p, z = p , we get $\mu\big(G(Tx_n,Tp,Tp)+\varphi(Tx_n)+2\varphi(Tp)\big)\le\mu(L(x_n,p,p))-\varphi(M(x_n,p,p))$ or $\mu\big(G(x_{n+1},Tp,Tp)+\varphi(x_{n+1})+2\varphi(Tp)\big)\le\mu(L(x_n,p,p))-\varphi(M(x_n,p,p))$ (11)

where $L(x_n,p,p)=\max\{G(x_n,p,p)+\varphi(x_n)+2\varphi(p),G(x_n,Tx_n,Tx_n)+\varphi(x_n)+2\varphi(Tx_n),G(p,Tp,Tp)+\varphi(p)+2\varphi(Tp),G(p,Tp,Tp)+\varphi(p)+2\varphi(Tp),\frac{1}{3}\{G(x_n,Tp,Tp)+\varphi(x_n)+2\varphi(Tp)+G(p,Tp,Tp)+\varphi(p)+2\varphi(Tp)+G(p,Tx_n,Tx_n)+\varphi(p)+2\varphi(Tx_n)\}$

Taking limit as n→∞,

$\lim_{k\to\infty} L(x_n,p,p)=$
$\max\{G(p,p,p)+\varphi(p)+2\varphi(p),G(p,Tp,Tp)+\varphi(p)+2\varphi(Tp),G(p,Tp,Tp)+\varphi(p)+$

$2\varphi(Tp), G(p,Tp,Tp) + \varphi(p) + 2\varphi(Tp),\ \frac{1}{3}\{G(p,Tp,Tp) + \varphi(p) + 2\varphi(Tp) + G(p,Tp,Tp) + \varphi(p) + 2\varphi(Tp) + G(p,Tp,Tp) + \varphi(p) + 2\varphi(Tp)\}$

$= G(p,Tp,Tp) + 2\varphi(Tp)$ and

$M(x_n, p, p) = \max\{G(x_n,p,p) + \varphi(x_n) + 2\varphi(p), G(p,Tp,Tp) + \varphi(p) + 2\varphi(Tp), G(p,Tp,Tp) + \varphi(p) + 2\varphi(Tp)\}$

Taking limit as n→∞,

$\lim_{k\to\infty} M(x_n,p,p) = \max\{G(p,p,p) + \varphi(p) + 2\varphi(p), G(p,Tp,Tp) + \varphi(p) + 2\varphi(Tp), G(p,Tp,Tp) + \varphi(p) + 2\varphi(Tp)\}$ $= G(p,Tp,Tp) + 2\varphi(Tp)$

Hence from (11), as n→∞ we get that $\mu\big(G(p,Tp,Tp) + 2\varphi(Tp)\big) \leq \mu(G(p,Tp,Tp) + 2\varphi(Tp)) - \varphi(G(p,Tp,Tp) + 2\varphi(Tp))$ which implies that $\varphi(G(p,Tp,Tp) + 2\varphi(Tp)) \leq 0$ and so that $G(p,Tp,Tp) + 2\varphi(Tp)$= 0 which gives $\varphi(Tp)$= 0 and Tp = p, showing that p is a fixed point of T. The uniqueness of fixed point follows from (1) .This completes the proof.

Corollary 2.2. Let (X,G) be a complete G-metric space and T be a self map of X satisfying $\mu(G(Tx,Ty,Tz) + \varphi(Tx) + \varphi(Ty) + \varphi(Tz)) \leq \mu(L(x,y,z)) - \varphi\big(L(x,y,z)\big)$ for all x,y,z in X,where L ,μ and φ are given as in Theorem 2.1. Then there exists a unique p in X such that p = Tp and $\varphi(p) = 0$.

Proof : Follows from the Theorem 2.1 by taking $M(x,y,z) = L(x,y,z)$.

Corollary 2.3. Let (X,G) be a complete G-metric space and T be a self map of X satisfying

$\mu(G(Tx,Ty,Tz) + \varphi(Tx) + \varphi(Ty) + \varphi(Tz)) \leq \mu(M(x,y,z)) - \varphi\big(M(x,y,z)\big)$ for all x,y,z in X,where M , μ and φ are given as in Theorem 2.1. Then there exists a unique p in X such that p = Tp and $\varphi(p) = 0$.

Proof : Follows from the Theorem 2.1 by taking $L(x,y,z) = M(x,y,z)$.

Corollary 2.4. Let (X,G) be a complete G-metric space and T be a self map of X satisfying $\mu(G(T^n x, T^n y, T^n z) + \varphi(T^n x) + \varphi(T^n y) + \varphi(T^n z)) \leq \mu(M(x,y,z)) -$

$\varphi\big(M(x,y,z\,)\big)$ for all x,y,z in X,where M , μ and φ are given as in Theorem 2.1. Then there exists a unique p in X such that p = Tp and $\varphi(p) = 0$.

Proof : Take $T^n = A$ in the Theorem 2.1. Then A has a unique fixed point,say p, in X. Now T^n p = Ap = p and $\varphi(p) = \varphi(T^n p) = \varphi(Ap) = 0$. Since T^{n+1} p = Tp , ATp $=T^n Tp = T^{n+1}p = Tp$, which shows that Tp is also a fixed point of A. Hence by uniqueness of the fixed point , we get Tp = p.

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